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A Hybrid Particle Swarm Optimization with Tabu Search for Optimizing Aid Distribution Route

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ABSTRACT

This paper explores the use of metaheuristic algorithms for the Multi-Depot Vehicle Routing Problem, a complex form of the Vehicle Routing Problem crucial in logistics. The study contributes to operational research, offering strategies for effective logistics management and underscores the significance of metaheuristic algorithms in tackling intricate optimization problems. The study focuses on optimizing vehicle routes from multiple depots, using a k-clustering technique for initial grouping. It examines algorithms like Particle Swarm Optimization, Artificial Bee Colony, Ant Colony Optimization, and a hybrid of Particle Swarm Optimization with Tabu Search. These algorithms are vital for efficient route planning in varied environments, with practical implications demonstrated in real-world logistics scenarios. The findings revealed the limitations of the Particle Swarm Optimization PSO algorithm and showed the improvement with Tabu Search. While, the resulting hybrid, Particle Swarm Optimization with Tabu Search, demonstrated remarkable improvements and stands out for its efficiency and reliability in Multi-Depot Vehicle Routing Problem, it underscored the potential of metaheuristic algorithms in solving Nondeterministic Polynomial Time -hard combinatorial problems.

Yardım Dağıtım Rotası Optimizasyonu için Tabu Arama ile Hibrit Parçacık Sürü Optimizasyonu

ÖZ

Bu makale, lojistikte hayati öneme sahip Araç Rotalama Probleminin karmaşık bir formu olan Çok Depolu Araç Rotalama Problemi için metasezgisel algoritmaların kullanımını araştırmaktadır. Çalışma, etkili lojistik yönetimi için stratejiler sunarak operasyonel araştırmaya katkıda bulunmakta ve karmaşık optimizasyon sorunlarının çözümünde meta-sezgisel algoritmaların öneminin altını çizmektedir. Çalışma, ilk gruplama için k-kümeleme tekniği kullanılarak birden fazla depodan araç rotalarının optimize edilmesine odaklanır. Çalışmada, Parçacık Sürü Optimizasyonu, Yapay Arı Kolonisi, Karınca Kolonisi Optimizasyonu ve Tabu Aramalı hibrit Parçacık Sürü Optimizasyonu gibi algoritmaların başarısı incelenmektedir. Bu algoritmalar, gerçek dünyanın değişen ortamlarındaki lojistik senaryoların pratik uygulamalarında verimli rota planlaması için hayati öneme sahiptir. Bulgular Parçacık Sürü Optimizasyonu algoritmasının sınırlamalarını ortaya çıkarmış ve Tabu Arama ile iyileştirme yoluna gidilmiştir. Ortaya çıkan hibrit Tabu Aramalı hibrit Parçacık Sürü Optimizasyonu, Çok Depolu Araç Rotalama Problemi 'de dikkate değer gelişmeler göstererek verimliliği ve güvenilirliğiyle öne çıkarken, Deterministik Olmayan Polinom Zamanında-zor kombinatoryal problemlerin çözümünde metasezgisel algoritmaların potansiyelinin altını çizmiştir.

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1. Introduction

The Vehicle Routing Problem (VRP), an NP-hard problem in combinatorial optimization and logistics, has been a significant focus of both academic and practical interest since G. Dantzig and J. Ramser introduced it in 1959 [1]. VRP poses a challenge in efficiently allocating vehicular resources for transportation. Research has shown its potential to help organizations optimize their transportation logistics, potentially reducing costs by 5% to 20% [2]. This optimization is not only financially beneficial; it also plays a crucial role in the strategic management of human and material resources, thereby enhancing overall operational efficiency. The significant expansion and rising popularity of home delivery e-commerce services such as Amazon, Dalsey, Hillblom, and Lynn (DHL), AliExpress have spurred researchers to intensively investigate this area. Due to a variety of practical constraints, VRP has evolved into several specialized forms. These include the Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Windows (VRPTW), Multi-Depot Vehicle Routing Problem (MDVRP), Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW), Vehicle Routing Problem with Pickup and Delivery (VRPPD), Split Delivery Vehicle Routing Problem (SDVRP), Periodic Vehicle Routing Problem (PVRP), and the Stochastic Vehicle Routing Problem (SVRP). Each of these variants has unique characteristics that set it apart from others, with complexity increasing as more constraints are added. This study focuses on the MDVRP, which involves managing multiple depots and various delivery locations. The efficient delivery of aid during natural disasters like earthquakes and floods, as well as the rapid growth of e-commerce, highlights the critical importance of optimizing delivery routes. Effective route optimization in MDVRP can significantly reduce transportation costs, improve delivery efficiency, and ensure timely assistance to those in need. However, existing algorithms, while offering solutions, often face limitations in terms of performance, reliability, and adaptability to real-world constraints. This study aims to address these shortcomings by exploring innovative approaches to tackle the complexity of the MDVRP. To this end, the paper introduces a novel hybrid algorithm called Particle Swarm Optimization with Tabu Search (PSO-TS), combining the strengths of Particle Swarm Optimization (PSO) and Tabu Search (TS). This combination balances global search capabilities with local refinement, aiming to overcome the limitations of traditional algorithms. The study presents a comprehensive comparison of four metaheuristic algorithms: Ant Colony Optimization (ACO), PSO, Artificial Bee Colony (ABC), and PSO-TS. These metaheuristic algorithms, recognized for addressing the complexities of MDVRP, are explored to identify optimal routes. The experimental results demonstrate the superior performance and reliability of the PSO-TS approach, achieving lower average costs and reduced variability in solutions. These findings underscore the importance of exploring innovative hybrid algorithms to tackle complex combinatorial optimization problems, particularly in critical contexts like aid distribution during natural disasters and the fast-paced world of e-commerce delivery.

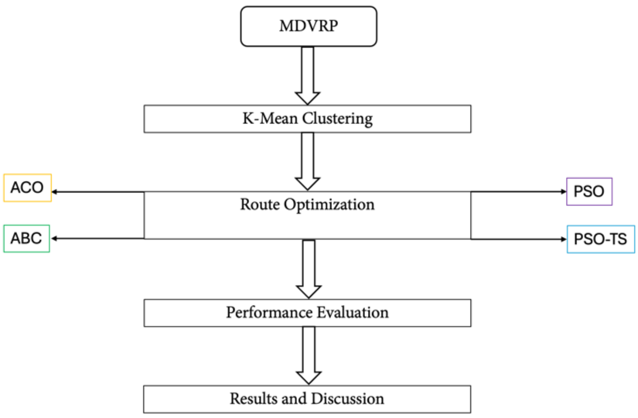

Figure 1. Block Diagram of the Research Process

This block diagram illustrates the methodology and key stages of the research, focusing on the development and evaluation of a hybrid PSO-TS algorithm for optimizing the MDVRP. To effectively tackle MDVRP, initially clustering methods is employed to group locations by their nearest depot. This strategy forms the foundation for applying metaheuristic algorithms to ascertain the most efficient routing for each depot. In this research, K-means clustering, an unsupervised machine learning technique, is utilized to categorize locations. Subsequently, the selected metaheuristic algorithms are deployed to determine the most efficient routes. In the realm of solving the MDVRP, various innovative algorithms have been proposed, each bringing unique approaches and methodologies to the forefront. Kicking off these advancements, Yongle He, Rong Xie, and Yanjun Shi [3] made a notable contribution with their development of a Tabu Search Algorithm incorporating Variable Cluster Grouping. This approach specifically targets MDVRP by intelligently grouping delivery locations into clusters, which are then optimized using a Tabu Search framework. This method significantly improves route efficiency and reduces overall transportation costs. In the middle of these developments, Matic Pintarič and Sašo Karakatič [4] employed PSO to tackle MDVRP. Their approach leverages the collective behavior observed in natural swarms, adapting it to optimize the routes in a multi-depot context. This method stands out for its ability to efficiently navigate the complex search space of MDVRP.

Close on their heels, Nafiz Mahmud and Md Mokammel Haque [5] applied a Genetic Algorithm (GA) for MDVRP solutions. Their approach mimics the process of natural selection, where the fittest routes survive and evolve over generations, leading to increasingly efficient routing solutions. Per Stodola and Jan Nohel [6] proposed an innovative solution using adaptive Ant Colony Optimization with Node Clustering for MDVRP. This method combines the robust optimization techniques of ant colony behavior with a strategic clustering of nodes, significantly enhancing route optimization and efficiency.

Toward the latter part of these advancements, Fernando Bernardes de Oliveira, Rasul Enayatifar, Hossein Javedani Sadaei, Frederico Gadelha Guimarães, and Jean-Yves Potvin developed [7] a cooperative coevolutionary algorithm for MDVRP. Their approach integrates different evolutionary strategies in a cooperative framework, enabling more effective problem-solving for complex routing scenarios. Lastly, the team comprising Zhaoquan Gu1 Yan, Zhu Yuexuan, Wang, Xiaojiang Du, and Mohsen Guizani [8] made a significant contribution by applying an Artificial Bee Colony algorithm to MDVRP. This approach models the foraging behavior of honeybees, offering a robust and efficient method for solving routing problems in a multi-depot environment.

Recent research has further expanded the complexity and application of MDVRP. Arishi and Krishnan [25] introduced a novel multi-agent deep reinforcement learning (MADRL) approach for solving the MDVRP, demonstrating its potential for dynamic routing. Soriano, Gansterer, and Hartl [26] highlighted the importance of profit fairness in MDVRP solutions, proposing a model and heuristic algorithm for equitable profit distribution. Wirawan and Suharjito [27] showcased the benefits of integrating geographical information systems (GIS) with MDVRP solutions for optimizing retail deliveries. Lim, Lee, and Singgih [28] explored the Multi-Depot Split-Delivery Vehicle Routing Problem (MDSDVRP), where deliveries can be split across multiple vehicles. Finally, Chen et al. [29] tackled a complex variant of the MDVRP, introducing the "Waitable Time-Varying Multi-Depot Green Vehicle Routing Problem" (WT-MDVRP) and proposing a genetic algorithm for solving it. These studies illustrate the ongoing advancements in MDVRP research, exploring different algorithmic approaches, addressing complex variants, and considering real-world constraints like time windows, profit fairness, and environmental impact.

Each of these methods represents a stride forward in optimizing solutions for the MDVRP, showcasing the diverse and innovative approaches researchers have employed to tackle this complex logistics challenge.

2. Method

The research methodology is designed to address the complexities of the MDVRP efficiently. Recognizing the inherent challenges in managing multiple depots and optimizing routes, the proposed approach combines the strengths of unsupervised machine learning and advanced optimization techniques. The aim of this study is to minimize the total travel distance, while effectively managing the distribution of resources from multiple depots.

2.1. K-Means clustering

The first phase involves grouping delivery locations into clusters based on their proximity to the nearest depots. The K-means clustering algorithm is employed for this purpose, a method widely recognized for its effectiveness in unsupervised machine learning. The objective in this phase is to minimize the within-cluster sum of squares, which essentially reduces the variance within each cluster.

Let:

- $D = \{d_1, d_2, \dots, d_m\}$ be the set of depots.
- $L = \{l_1, l_2, \dots, l_n\}$ be the set of delivery locations.
- k be the number of clusters, typically equal to m, the number of depots.
- $C = \{c_1, c_2, \dots, c_k\}$ be the set of cluster centroids.

The K-means clustering objective is to minimize the sum of squared distances between delivery locations and their nearest centroid. The centroids are initially chosen randomly or based on specific criteria. Each delivery location l_i is assign to cluster c_j such that the Euclidean distance $d(l_i, c_j)$ is minimized (Equation 1).

$$d(l_i, c_j) = \sqrt{\sum_{p=1}^{n}(l_{ip} - c_{jp})^2} \tag{1}$$

where l_i and c_{jp} are the coordinates of the delivery location and centroid in the p-th, dimension. After assigning all locations, centroid of the clusters is updated as shown in Equation 2.

$$c_j = \frac{1}{|S_j|}\sum_{i \in S_j} l_i \tag{2}$$

where S_j is the set of locations in cluster j.

2.2. Route optimization

Once clusters are formed, the next phase is route optimization. This involves solving the MDVRP under a set of constraints to minimize the total distance traveled while ensuring efficient resource utilization. The Multi-Depot Vehicle Routing Problem (MDVRP) can be effectively modeled as a network to facilitate analysis and solution development. In this context, the network is represented by a graph G = (V, E) where V denotes the set of vertices or nodes, and E represents the set of edges or paths.

- V: The set of vertices in the network, denoted as V, includes all the depots and delivery locations. This can be represented as: $V = D \cup L$ here $D = \{d_1, d_2, \dots, d_m\}$ is the set of m depots, and $L = \{l_1, l_2, \dots, l_n\}$ is the set of n delivery locations.
- E: The edges in the network, denoted as E, symbolize the feasible routes or connection between vertices. Defined as: $E = \{(i, j) | i, j \in V \text{ and } i \neq j\}$
- $C = \{C_1, C_2, \dots, C_m\}$: Clustering formed post K-means clustering, where C_d is the cluster of locations associated with depot d.
- K_d: Set of vehicles allocated to depot d, with $|K_d|$ being the number of vehicles.
- Q_k: Capacity of vehicle k.
- q_i: Demand at location i.
- d_{ij}: Distance between locations i and j.

The decision variables are as defined below,

$$x_{ijd} = \begin{cases} 1, & taken \\ 0, & not\ taken \end{cases} \tag{1}$$

where binary decision variable, 1 if a vehicle from depot d travels from location i to location j; 0 otherwise.

The objective function is defined in Equation 2,

$$Min \sum_{d \in D} \sum_{k \in K_d} \sum_{i,j \in C_d, i \neq j} d_{ij} \cdot x_{ijd} \tag{2}$$

where several constraints are considered as defined in defined in equations 3 to 5.

- Vehicle Capacity Constraint

$$\sum_{i \in C_d} q_i \cdot x_{ijd} \leq Q_k \quad \forall k \in K_d, \ \forall d \in D \tag{3}$$

The delivery capacity of each vehicle should not exceed the maximum capacity.

- Customer Visit Constraints

$$\sum_{j \in C_d, j \neq i} x_{ijd} = 1 \quad \forall l_i \in C_d, \forall d \in D \tag{4}$$

Only one vehicle per customer's location.

- Depot Start and End Constraints

$$\sum_{i \in C_d} x_{dik} = 1, \sum_{j \in C_d} x_{kjd} = 1 \quad \forall k \in K_d, \forall d \in D \tag{5}$$

Figure 1 demonstrates a sample MDVRP.

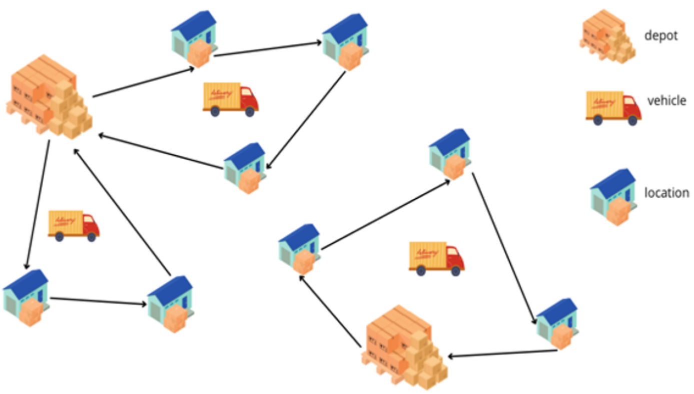

Figure 2. MDVR example

2.3. Ant colony optimization

ACO is a robust metaheuristic algorithm inspired by the natural foraging behavior of ants. Developed by M. Dorigo and colleagues [9], ACO has been widely recognized for its effectiveness in solving complex optimization problems, particularly in logistics and routing. The algorithm's foundations lie in mimicking how ants find the shortest path between their colony and food sources, a process driven by pheromone trails and collaborative efforts. This approach is termed the "Artificial Ant Colony Algorithm", a name chosen to differentiate it from natural ant systems.

In the context of the MDVRP, ACO is particularly effective. The MDVRP, akin to the Travelling Salesman Problem (TSP) but with added complexity, involves determining the most efficient routes for a fleet of vehicles operating from multiple depots. By mimicking the ants' method of iteratively refining their paths based on pheromone trails, ACO allows for a dynamic and adaptive search for the most efficient routing solutions in MDVRP scenarios. on paths they traverse.

Key Components of ACO

- Ant Agents: Each ant in the algorithm represents a potential solution, i.e., a sequence of routes for the vehicles.
- Pheromone Trails(τ): Ants lay down pheromone trails (τ_{ij}) on paths they traverse. These trails, which decay over time, guide subsequent ants' decisions: $\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \Delta_{\tau_{ij}}$

where p is the pheromone evaporation rate, and $\Delta_{\tau_{ij}}$ is the amount of pheromone deposited.

- Heuristic information (η): This is typically the inverse of the distance (d_{ij}) between location two locations i and j : $\eta_{ij} = \frac{1}{d_{ij}}$. It represents the desirability of a route.

Table 1 details the key steps and mathematical elements of the ACO process in the context of the clustered MDVRP.

Table 1. ACO for MDVRP process

Stage	Description	Formula
Preprocessing using K-means Clustering	Group delivery locations into clusters based on proximity to the nearest depot, forming clusters C_d for each depot d.	N/A
Initialization	Set initial pheromone levels on paths within clusters.	Pheromone τ_{ij} levels initialized to a small positive value.
Route Construction	Ants build routes within each cluster C_d, starting from the associated depot. Selection of the next location based on pheromone and heuristic information.	Probability p_{ij}^k of choosing the next location j from i by ant k, $p_{ij}^k = \frac{(\tau_{ij}^\alpha)\cdot(\eta_{ij}^\beta)}{\sum(\tau_{ij}^\alpha \cdot \eta_{ij}^\beta)}$
Local Pheromone Update	Adjust pheromone levels on individual paths post visit by ant, promoting exploration.	Local update rule: $\tau_{ij} \leftarrow (1-\phi)\cdot \tau_{ij} + \phi \cdot \tau_0$
Global Pheromone Update	After all ants' complete routes, globally update pheromones on successful paths. Incorporate pheromone evaporation to avoid premature convergence.	Global update rule: $\tau_{ij} \leftarrow (1-\phi)\cdot \tau_{ij} + \Delta_{\tau_{ij}}$
Iterative Optimization and Convergence	Repeat the process for multiple iterations until a stopping criterion is met (fixed number of iterations)	N/A

2.4. Artificial bee colony

The ABC algorithm, initially proposed by Derviş Karaboğa in 2005 through his pioneering work titled "An Idea Based on Honeybee Swarm for Numerical Optimization" (Technical Report-TR06, Erciyes University), is a bio-inspired optimization strategy that has **gained** widespread recognition in various scientific and engineering fields [10]. This algorithm, which draws inspiration from the foraging patterns of honeybees, is a stochastic, population-based method that has been effectively applied in diverse areas [11]. The structure of the ABC algorithm includes three types of bees: employed bees, onlookers, and scouts. Each group plays a specific role, collaboratively contributing to the search for optimal solutions [12]. This unique collaborative mechanism enables the ABC algorithm to maintain a balance between exploration and exploitation of solutions [13]. Notably, the ABC algorithm has been utilized extensively in numerical optimization and engineering. It has also been applied in various other domains, such as cancer classification [14], SAR image segmentation [15], and the estimation of induction motor parameters [16]. Its ability to robustly optimize numerical problems has been well-documented [17]. To enhance its capabilities, various modifications have been introduced, including search space division and a disruptive selection strategy [18]. Moreover, to address specific optimization challenges, several variants and hybrid versions of the ABC algorithm have been developed. These include the enhanced memetic ABC, a global ABC variant that incorporates crossover and Tabu Search techniques, and a hybrid ABC integrating variable neighborhood search with a memory mechanism. Such analyses underscore the significance and efficiency of ABC in the realm of optimization algorithms, echoing its foundational roots laid by Karaboğa. For the MDVRP problem, ABC is applied to a single VRP, utilizing K-means clustering to assign specific locations to each depot. Algorithm 1 shows the process of ACO for MDVRP.

Algorithm 1: Artificial Bee Colony for MDVRP

Depot Clustering

1

*Apply **k-means** clustering to assign location to the nearest depot(s).*

***Output**: Clustered groups of locations for each depot.*

Initialize Solutions

For each depot, generate initial solutions (food source) and calculate their fitness.
Set iteration counter $\boldsymbol{i} = \mathbf{0}$

Iteration Optimization

***While** i < Max_iteration:*

For** each depot**:

ABC for single VRP

Generate initial solutions as food sources for the depot and calculate fitness.
Assign employed bees to each food source.

Employed bees perform local search to improve their assigned solutions.
Follower and scout bees explore new solutions.

Record the best () solution for each depot.

Depot Combination

Calculate additional cost of combining best solutions from

different depots.

*Increment iteration counter (**i←i + 1**)*

Repeat Until Convergence

Continue the iterative process until the maximum number of iterations is reached.

Let's consider the following scenario of the Multi-Depot Vehicle Routing Problem (MDVRP), characterized by two depots, D = $\{d_1, d_2\}$, and a set of locations, L = $\{l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8, l_9\}$. The initial step involves assigning each location to the nearest depot. This process is demonstrated in Figure 2, where the locations $\{l_1, l_2, l_3, l_8, l_6\}$ are grouped into a cluster with depot d1, while the locations $\{l_7, l_9, l_5, l_4\}$ form a cluster with depot d2.

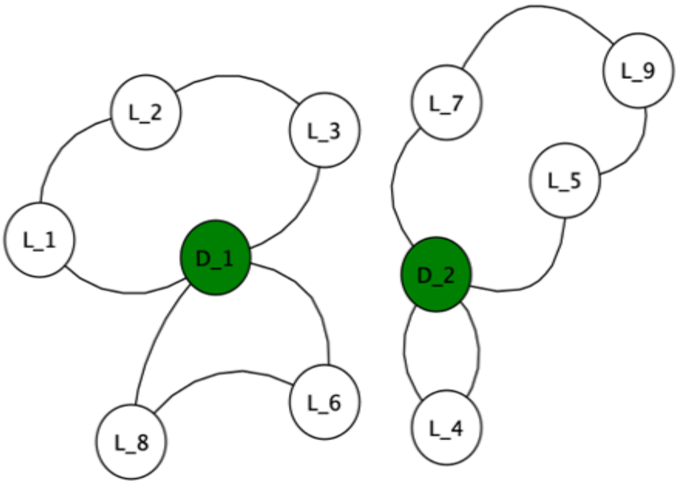

Figure 3. MDVRP with clusters

The solutions, referred to as food sources, must then be evaluated. The assessment of each solution is facilitated by calculating its fitness, which can be defined as given in Equation 6:

$$F(Solution) = \frac{1}{1+TotalDistance(Solution)} \quad (6)$$

where the *TotalDistance* represents the sum of the distances of all routes in the solution.

During the phase for employed bees, these bees will seek out new potential food sources in the vicinity of their current one. Therefore, a set of local search strategies is established for the "employed bees" to identify superior solutions. Within the context of local search strategies used by employed bees in the Artificial Bee Colony (ABC) algorithm, three common techniques including random insert, swap, and inverse are explained as follows:

- Random insert

This method involves randomly selecting a location from a route and inserting it into another position within the same route (Figure 4).

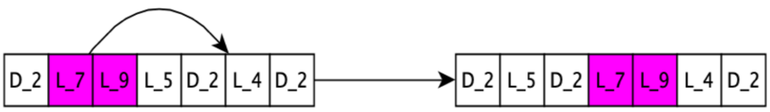

Figure 4. Insertion

- Random Swap

The swap method selects a set of locations on a route and swaps their positions (Figure 5).

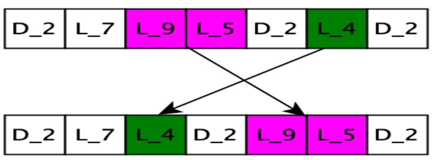

Figure 5. Swap

- Random Inverse

This method takes a sequence of locations within a route and inverts their order (Figure 6).

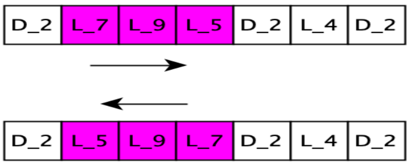

Figure 6. Inverse

2.5. Particle Swarm Optimization

PSO is a population-based stochastic optimization algorithm, inspired by the social behavior of bird flocking or fish schooling. It was first introduced by Dr. Eberhart and Dr. Kennedy in 1995 [19]. PSO is a metaheuristic algorithm that has been widely applied to solve various optimization problems in engineering, computer science, economics, and other fields. The basic concept of PSO involves simulating the social behavior of a flock of birds searching for food.

In PSO, a population of potential solutions, called particles, move through the search space. Each particle adjusts its position based on its own experience and the experience of its neighbors. The movement of particles is guided by their own best-known position and the best-known position of the entire swarm. This collective behavior enables the particles to converge toward the optimal solution over iterations.

One of the key components of PSO is the fitness function, which evaluates the quality of a potential solution. The particles adjust their positions based on the value of the fitness function, aiming to minimize or maximize it, depending on the nature of the optimization problem. PSO has several parameters that need to be carefully tuned, such as the cognitive and social parameters, inertia weight, and the size of the swarm. These parameters significantly influence the convergence behavior and performance of the algorithm. Numerous variants and modifications of PSO have been proposed to enhance its performance and address specific challenges. These include adaptive PSO, multi-objective PSO, hybrid PSO, and constrained optimization PSO, among others.

Like the previous algorithms, PSO is applied for a single VRP because K-means clustering is used, and each depot has its own locations. Algorithm 2 shows the process.

Algorithm 2: Particle Swarm Optimization for MDVRP

1 ***Depot Clustering***
2   *Apply **k-means** clustering to assign location to the nearest depot(s).*
3   ***Output**: Clustered groups of locations for each depot.*
4 ***PSO for each cluster:***
5   ***Initialization***
6   ***Number of Particles:** Set N, the number of particles in the swarm.*
7   ***Particles Positions and Velocities:** Randomly initialize the position $\boldsymbol{X_i}$ and velocity $\boldsymbol{V_i}$ for each particle i.*
8   ***Best known Positions:** Initialize the personal best position $\boldsymbol{P_{best}}$ for each particle i and global best position $\boldsymbol{G_{best}}$ across all particles.*
9   ***For each iteration:***
10   ***For each particle:***
11   ***Fitness Calculation:** Calculate the fitness value F_i of the particle's current position(solution).*
12   ***Update Personal Best: If** F_i is better than the fitness of $\boldsymbol{P_{best}}$, then set $\boldsymbol{P_{best}} = \boldsymbol{X_i}$*
13   ***Update Global best:** Find the particle with the best fitness in the current iteration and update $\boldsymbol{G_{best}}$ if it's better than the current*

$\boldsymbol{G_{best}}$
Update Velocity:

$$V_i = \omega \cdot V_i + c_1 \cdot r_1 \cdot (P_{best} - X_i) + c_2 \cdot r_2 \cdot (G_{best} - X_i)$$

Where ω is the inertia weight, c_1 and c_2 are cognitive and social coefficients respectively, and r_1, r_2 are random numbers in $[0,1]$
Position Update: $X_i = X_i + V_i$
Record the solution (route and cost for the cluster)
Combine solutions and calculate total cost
Aggregate the solutions from all clusters
Calculate the total cost (sum of costs from all clusters solutions)
Output*: Return the combined routing solution and the total cost.*

2.6. Proposed Hybrid PSO

Upon applying the three algorithms, it was noted that PSO yielded the lowest cost. To further optimize this, Tabu Search was integrated, with details available in the experimental results section. Tabu Search, recognized for its effectiveness in diverse optimization challenges, has shown success in areas like scheduling, vehicle routing, and combinatorial optimization. It has also been valuable in solving classical problems like the traveling salesman [1]. Furthermore, Tabu Search has proven effective compared to existing algorithms for MaxMeanDP and the vehicle routing problem [2], [3]. The algorithm's iterative local search technique starts from an initial solution and explores the solution space to find optimal solutions [4]. Additionally, Tabu Search has been described as a higher-level heuristic, guiding other methods to escape local optima [1].

The PSO-TS approach for solving the Multi-Depot Vehicle Routing Problem (MDVRP) is an innovative hybrid algorithm that combines the strengths of Particle Swarm Optimization (PSO) and Tabu Search (TS). This method first utilizes PSO to explore a broad solution space efficiently and then applies the TS technique to refine these solutions. This combination aims to balance global and local search capabilities, effectively addressing the complexities of MDVRP to find optimized routing solutions. The proposed Algorithm 3 uses operators like 1-swap, 1-insert, and 2-opt to create neighboring solutions.

Algorithm 3: PSO- TS for MDVRP

PSO
Refine the returning $\boldsymbol{G_{best}}$ *PSO solution*
TS
For the current set of routes
Generate Initial Solution
Neighborhood Structure
Use operators like 1-swap, 1-insert, and 2-opt to create neighboring solution.
Tabu List Management
Maintain a Tabu List to record history and avoid local optima.
Prohibit certain moves based on the tabu list

3. Experimental Results and Discussion

All experiments were conducted on a MAC OS operating system with an Intel Core i7 processor. All algorithms were implemented using Python 3.1, utilizing Jupyter Notebook and Spyder as the Integrated Development Environments (IDEs). The problem data, sourced from a repository dedicated to MDVRP problem datasets [5], comprises a text file with 50 locations and 4 depots. Each location includes coordinates (x, y), vehicle capacity (80), the number of vehicles (maximum 4 per depot), and demand per location.

To better understand the data, a program was developed to convert the text file into a JSON format (Figure 7), facilitating easier data utilization.

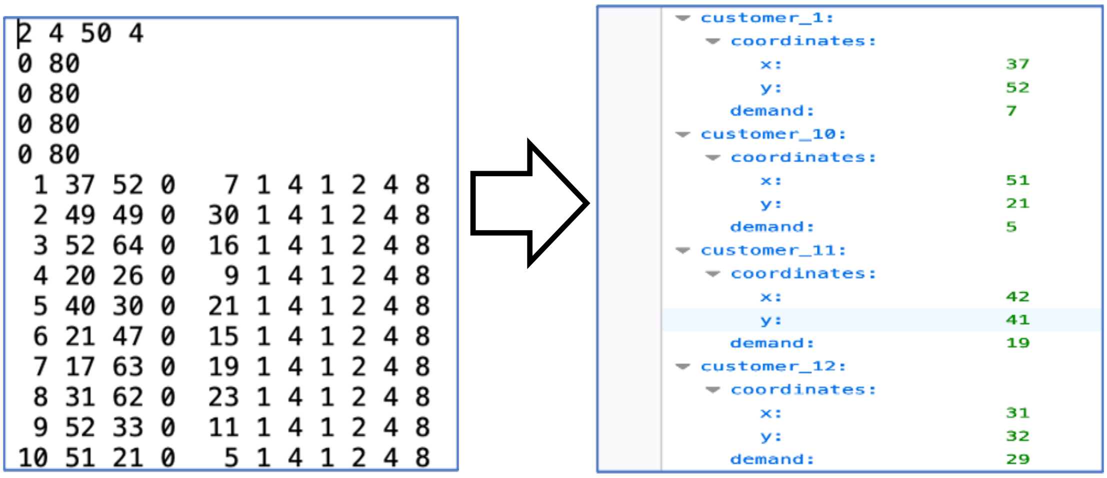

Figure 7. Converting txt file to JSON

Initially, K-means clustering was applied to group locations by their nearest depot. Figure 8 shows the results of K-means clustering.

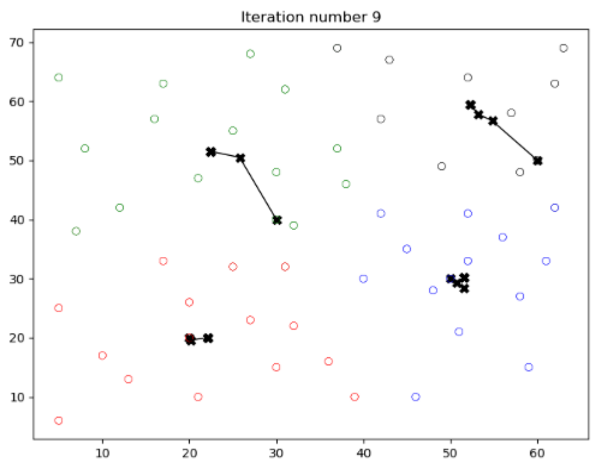

Figure 8. K-Means clustering

After 9 iterations, the locations have been grouped by the nearest depots as shown in Table 2. Figure 9 demonstrates how ABC algorithm behaves.

Table 2. Clusters

C	Depots	Locations
c_1	51	4, 12, 13, 15, 17, 18, 19, 37, 40, 41, 42, 44, 45, 47
c_2	52	1, 6, 7, 8, 14, 23, 24, 25, 26, 27, 32, 43, 46, 48
c_3	53	5, 9, 10, 11, 16, 21, 30, 33, 34, 38, 39, 49, 50
c_4	54	2, 3, 20, 22, 28, 29, 31, 35, 36

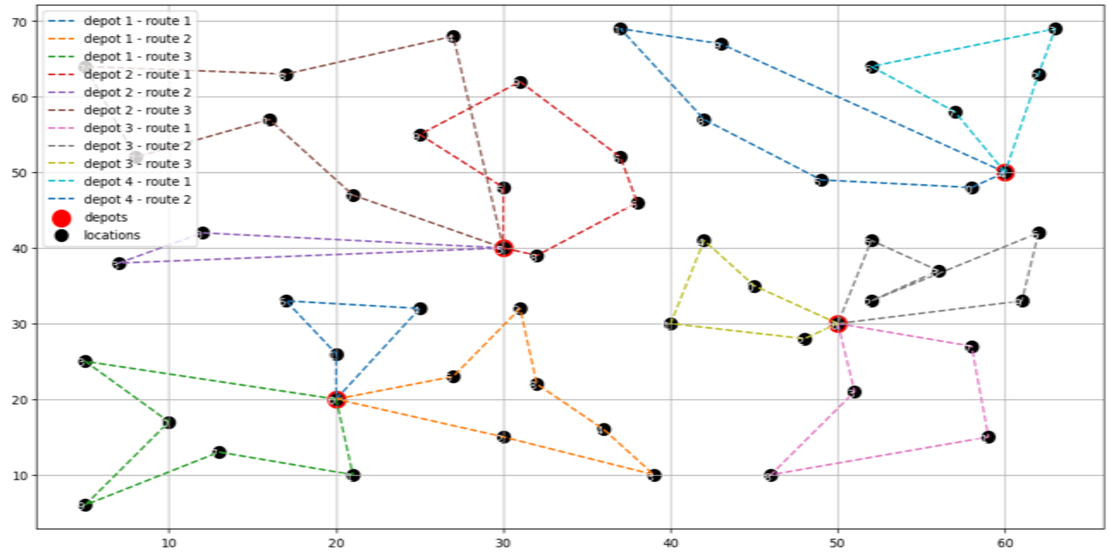

Figure 9. ABC for MDVRP

Table 3 shows each route along with the corresponding locations. After the algorithm, it was observed that each cluster attempted to achieve the optimum cost while adhering to the constraints previously outlined in the network's definition. The ABC algorithm was chosen to demonstrate how the total cost was computed from clusters.

Table 3. Clusters with routes

Cluster	Routes		ABC Cost
	1	51 – 45 – 17 – 47 – 51	
C1	2	51 – 15 – 4 – 19 – 13 – 44 – 42– 51	164.14
	3	51 – 12 – 40 – 37 – 18 – 41 – 51	
	1	52 – 43 – 27 – 48 – 7– 46 – 26 – 52	
C2	2	52 – 8 – 24 – 52	198.39
	3	52 – 25 – 6 – 32 – 23 – 14 – 1– 52	
	1	53 – 9 – 30 – 38 – 21– 53	
C3	2	53 – 33 – 16 – 5 – 49 – 11 – 53	148.09
	3	53 – 34 – 10– 50 – 39 – 53	
C4	1	54 – 3 – 2 – 35 – 31 – 54	113.55
	2	54 – 22 – 29– 30 – 36 – 28 – 54	
Total Cost		624.17	

All algorithms were run 30 times. For comparison of different algorithms under identical conditions, each algorithm was run with the same number of function evaluations, set as 100. The initial best state of each algorithm varies, as it must adhere to defined constraints and is determined randomly based on each algorithm's ability to approach the optimal solution. Figure 10 illustrates the convergence curves of all algorithms for one run out of 30. It was observed that, after a certain number of iterations, the cost for each algorithm stabilized and became constant. Furthermore, the figure demonstrates that the cost for PSO-TS starts at the same point as that for PSO, indicating an initial similarity in performance between these two approaches.

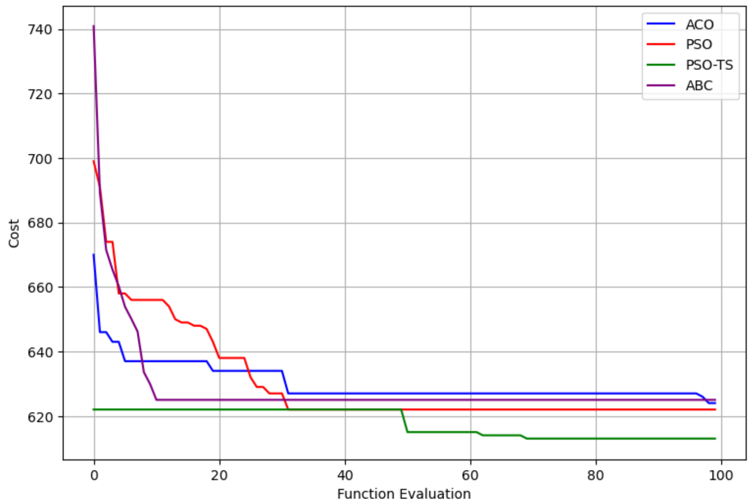

Figure 10. Convergence graph

Figure 11-13 shows the total costs over different runs for all algorithms. After 30 runs, the minimum costs obtained were 609.27 for ABC, 613.00 for PSO and 619.00 for ACO.

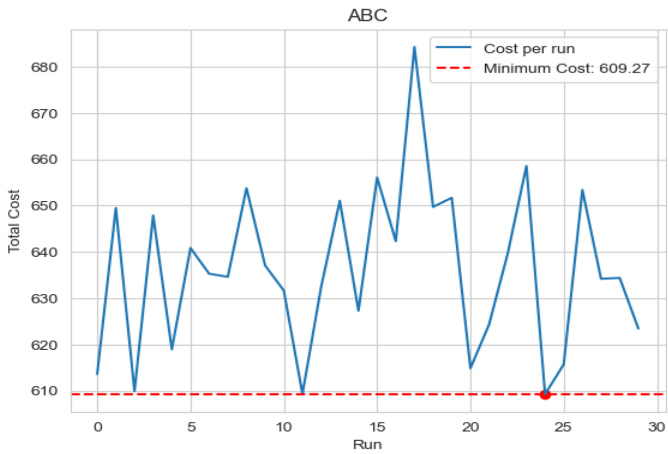

Figure 11. ABC for MDVRP

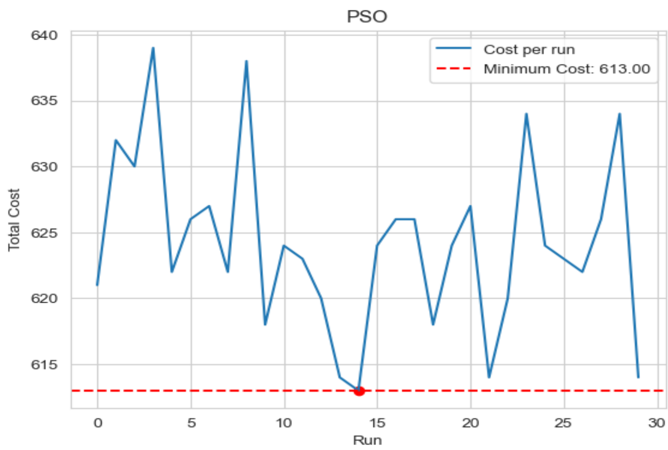

Figure 12. PSO for MDVRP

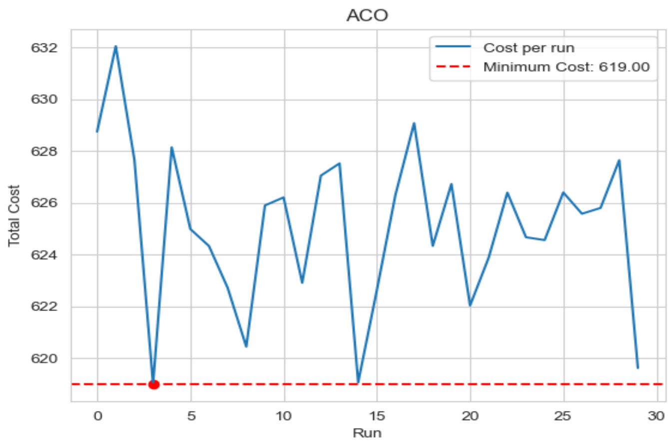

Figure 13. ACO for MDVRP

Upon recognizing that the ABC algorithm yielded the lowest cost, Tabu Search was used to improve the quality of the best solution obtained by PSO. The hybrid PSO-TS algorithm successfully reached an optimum cost of 608.00 (Figure 14).

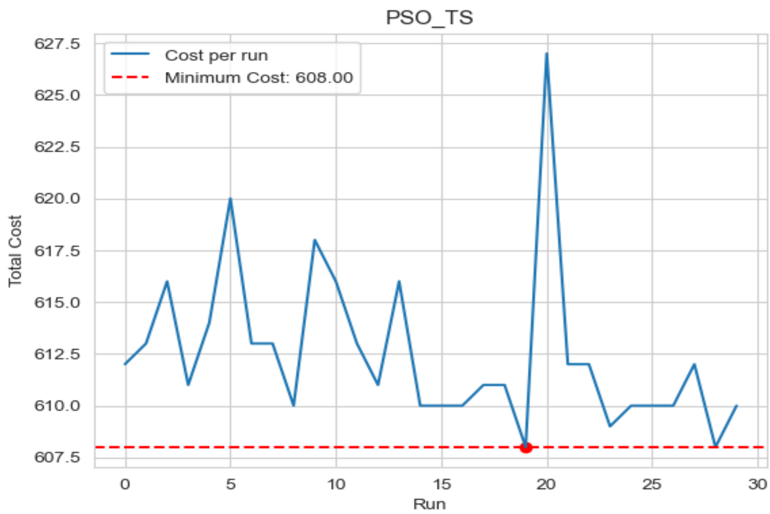

Figure 14. PSO-TS for MDVRP

In evaluating the performance of algorithms discussed in this study, a suite of statistical metrics was leveraged, notably the mean and standard deviation (std) and quartile values (25%, 50%, 75%). The mean was calculated as the sum of all costs divided by the number of runs. The standard deviation is computed as the square root of the average squared differences from the mean quantifies the spread or variability of the cost values around the mean. The quartile values further dissect the obtained costs into four equal parts: the 25% quartile (first quartile) denotes the median of the costs's lower half, the 50% quartile (median) effectively splits the costs, and the 75% quartile (third quartile) represents the median of the costs's upper half. These measures provide a comprehensive snapshot of costs distribution. Together, they are critical for a thorough assessment of algorithm performance offering a nuanced understanding of an algorithm's operational efficiency and robustness.

Table 4. Performance comparison of the algorithms for MDVRP

	Algorithm Performance comparison for MDVRP			
	PSO	**ACO**	**ABC**	**PSO-TS**
Mean	624.16	625.07	636.17	612.53
Min	613.00	619.00	609.26	608.00
25%	620.25	623.15	623.67	610.00
50%	624.00	625.68	634.95	611.50
75%	626.75	626.96	649.67	613
Max	639.00	632.28	684.28	627.00
Std	6.70	3.07	17.70	3.93

Table 4's comparison of algorithms for solving the Multiple Depot Vehicle Routing Problem (MDVRP) reveals how Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Artificial Bee Colony (ABC), and PSO with Tabu Search (PSO-TS) perform over 30 runs. PSO-TS shines as the top choice, delivering the lowest average cost of 612.53 and showing great reliability with a small standard deviation of 3.93. This means PSO-TS can consistently find good solutions without much change in performance. It also reached the lowest cost of 608.00, proving its effectiveness in solving MDVRP problems. PSO also does well, with an average cost of 624.16 and some variability, shown by a standard deviation of 6.70. Its lowest cost was 613.00, showing it can find low-cost solutions but not as reliably as PSO-TS. ACO's average cost is a bit higher than PSO's at 625.07, but it's the most consistent of the traditional algorithms, with the least variability at 3.07. However, its lowest cost is 619.00, suggesting it might struggle to find the lowest costs possible. ABC, on the other hand, has the most variation at 17.70 and the highest average cost of 636.17, indicating it might not always offer efficient solutions. But its minimum cost of 609.26 is competitive, showing it sometimes finds low-cost solutions despite its general inconsistency. Looking at the quartile values, PSO-TS consistently offers lower costs as 613, especially noticeable at its 75th percentile, showing uniform high performance. ABC's wide range, particularly its 75th percentile up to 649.67, indicates it often ends up with higher-cost solutions, even though it can occasionally find low-cost ones. In summary, PSO-TS stands out for its efficiency and reliability in MDVRP, making it a preferable option. While PSO and ACO provide solid, less variable solutions, ABC's variability suggests it's more of a gamble, capable of finding low-cost solutions but also risking higher-cost outcomes.

This study contributes to the growing body of research on solving the Multi-Depot Vehicle Routing Problem (MDVRP) using metaheuristic algorithms. Our findings highlight the efficiency and reliability of the hybrid PSO-TS approach, offering a promising solution compared to traditional algorithms like ACO, PSO, and ABC, particularly for optimizing aid delivery during natural disasters and e-commerce deliveries. However, understanding how our study fits within the broader landscape of MDVRP research is crucial. This discussion delves into related works, emphasizing the key similarities and differences between our approach and existing methods.
While various algorithms have been proposed to solve MDVRP, a few stand out in comparison to our study:

- Tabu Search with Variable Cluster Grouping [3]: This study, similar to ours, incorporates clustering, but it utilizes a Tabu Search algorithm. Their focus is on dynamically grouping delivery locations, while our approach utilizes K-Means clustering upfront and focuses on improving PSO performance with TS. This dynamic clustering strategy may be more advantageous in situations where locations are constantly changing or evolving, but our approach offers a simpler and potentially faster solution for static scenarios.
- Particle Swarm Optimization [4]: This work also uses PSO, but without the integration of Tabu Search. While our study demonstrates the improvement achieved by combining PSO with TS, their work focuses on the PSO algorithm's inherent ability to solve MDVRP. This comparison highlights the strengths of integrating a local search technique, like TS, to refine the solutions obtained by PSO.
- Genetic Algorithm [5]: This study applies a Genetic Algorithm to MDVRP, relying on natural selection principles to optimize routes. While GA is another effective metaheuristic approach, our study focuses on PSO and explores the benefits of hybridization with Tabu Search. This comparison emphasizes the diverse range of metaheuristic approaches for MDVRP and highlights the potential of hybridizing techniques for enhanced performance.

- Adaptive Ant Colony Optimization [6]: This work combines ACO with node clustering, offering a robust approach. While their focus is on adapting ACO for MDVRP, our research focuses on the potential of PSO and the enhancements achievable through hybridization with TS. This comparison demonstrates the versatility of ACO and its adaptability to different problem scenarios.
- Cooperative Coevolutionary Algorithm [7]: This method uses a cooperative evolutionary framework for MDVRP. While their approach emphasizes cooperation among evolutionary strategies, our study focuses on the combination of PSO and Tabu Search for improved optimization. This comparison highlights the advantages of different evolutionary strategies and the potential for combining them to tackle complex optimization problems.
- Artificial Bee Colony Algorithm [8]: This study uses ABC to solve MDVRP, drawing inspiration from the foraging behavior of honeybees. Although ABC offers a robust optimization approach, our study focuses on PSO and the benefits of integrating TS. This comparison underlines the variety of bio-inspired optimization algorithms available and emphasizes the ongoing research exploring their potential in solving complex problems like MDVRP.

While the proposed PSO-TS algorithm demonstrates promising results for solving the MDVRP, it's important to acknowledge certain limitations of this study:

- Dataset Scope: The study was conducted on a specific dataset with a limited number of depots and delivery points. The performance of the algorithm may differ when applied to larger and more complex real-world datasets with a greater number of locations and more intricate constraints.
- Simplified Scenario: The model currently does not account for real-world complexities such as traffic conditions, varying road types, or potential disruptions. Including these factors in future iterations would enhance the algorithm's practical applicability and make the results more relevant to real-world scenarios.
- Algorithm Optimization: The PSO-TS algorithm was optimized for the specific dataset used in this study. Further research is needed to assess its performance and potentially adapt the algorithm's parameters for other datasets or scenarios.
- Focus on Cost Optimization: While cost is a crucial factor in MDVRP, the algorithm's primary focus is on minimizing the total travel distance. Future research could explore incorporating additional objectives such as minimizing vehicle usage, reducing delivery time, or prioritizing routes based on urgency or sensitivity.

4. Conclusion

In this study, three distinct algorithms ACO, PSO, and ABC were analyzed to tackle the complexities of the Multi-Depot Vehicle Routing Problem (MDVRP). The findings revealed the limitations of the PSO algorithm and prompted development with Tabu Search. The resulting hybrid, PSO-TS, demonstrated remarkable improvements, underscoring the potential of metaheuristic algorithms in solving NP-hard combinatorial problems. Future research should focus on incorporating real-world factors, including traffic dynamics, fuel considerations, and additional logistical constraints, into the MDVRP model. This paper adds to the existing body of literature by showcasing the applicability of these advanced algorithms in progressively challenging scenarios, paving the way for more adaptive, robust, and efficient solutions in the field of complex problem-solving.

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